

Warm Up # 2- 4

Use: $y - f(c) = m(x - c)$

For a line tangent to $f(x)$ at $(c, f(c))$, $m = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$

1. Find the line tangent to $y = x^2 - 4$ at $x = 3$.
2. Find the line tangent to $y = x^3 + 2x^2 - 5x + 1$ at $x = -1$

No Warm Up tomorrow.

Staple and turn in:

Week 2 Classwork

Warm Up on top

Tan Limits WS

HW Questions: p. 92

In Exercises 1–20, find the given limit (if it exists).

$$1. \lim_{x \rightarrow 2} (5x - 3) \quad 3. \lim_{x \rightarrow 2} (5x - 3)(3x + 5) \quad 5. \lim_{t \rightarrow 3} \frac{t^2 + 1}{t}$$

$$7. \lim_{t \rightarrow -2} \frac{t + 2}{t^2 - 4} \quad 9. \lim_{x \rightarrow 0} \frac{[1/(x + 1)] - 1}{x} \quad 11. \lim_{x \rightarrow -1} \frac{x^3 + 1}{x + 1} \quad \text{factor}$$

$$13. \lim_{x \rightarrow 0^+} \left(x - \frac{1}{x^3} \right) \quad 15. \lim_{x \rightarrow -2} \frac{2x^2 + x + 1}{x + 2} \quad 17. \lim_{x \rightarrow -1} \frac{x + 1}{x^3 + 1} \quad \text{factor}$$

$$19. \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x + 1}$$

graph it
or look at
table

$$\frac{1(x+1)}{(x+1)(x^2-x+1)}$$

$$9. \lim_{x \rightarrow 0} \frac{[1/(x + 1)] - 1}{x}$$

$$\frac{1}{x} \left[\frac{1}{x+1} - 1 \left(\frac{x+1}{x+1} \right) \right]$$

$$= \lim_{x \rightarrow 0} \frac{1}{x} \left(\frac{1 - x - 1}{x + 1} \right)$$

$$= \lim_{x \rightarrow 0} \frac{1}{x} \cdot \frac{-x}{x+1}$$

$$= \lim_{x \rightarrow 0} \frac{-1}{x+1}$$

$$= -\frac{1}{1}$$

$$= \boxed{-1}$$

$$13. \lim_{x \rightarrow 0^+} \left(x - \frac{1}{x^3} \right)$$

$$\begin{array}{l} 0^+ - \frac{1}{0^+} \\ \text{sm pos} \\ 0^+ - \infty \\ -\infty \end{array}$$

21. Estimate the limit

$$\lim_{x \rightarrow 1^+} \frac{\sqrt{2x+1} - \sqrt{3}}{x-1}$$

by completing the following table.

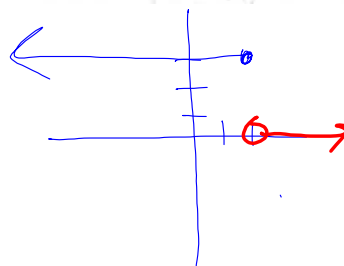
x	1.1	1.01	1.001	1.0001
$f(x)$				

23. Evaluate the limit in Exercise 21 by rationalizing the numerator.

In Exercises 25–30, determine whether the given limit statement is true or false.

$$25. \lim_{x \rightarrow 0} \frac{|x|}{x} = 1 \quad 27. \lim_{x \rightarrow 2} f(x) = 3, \text{ where } f(x) = \begin{cases} 3, & x \leq 2 \\ 0, & x > 2 \end{cases}$$

$$29. \lim_{x \rightarrow 0^+} \sqrt{x} = 0$$



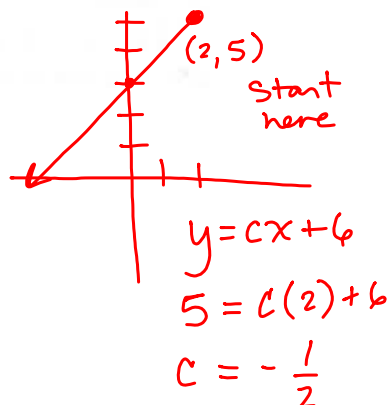
In Exercises 31–38, determine the intervals on which the given function is continuous.

$$31. f(x) = \llbracket x + 3 \rrbracket \quad 33. f(x) = \begin{cases} \frac{3x^2 - x - 2}{x - 1}, & x \neq 1 \\ 0, & x = 1 \end{cases}$$

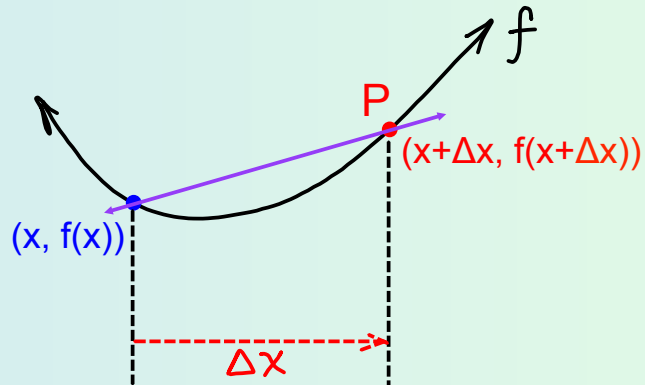
$$35. f(x) = \frac{1}{(x-2)^2} \quad 37. f(x) = \frac{3}{x+1}$$

39. Determine the value of c so that the following function is continuous on the entire real line.

$$f(x) = \begin{cases} x + 3, & x \leq 2 \\ cx + 6, & x > 2 \end{cases}$$



The Limit Definition of the Derivative.



$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Provided the limit exists.

We also already know the Alternate Form of the Derivative:

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

We have been using it to find the derivative at a known value $x = c$.

(Finding the slope of a tangent at $x = c$)

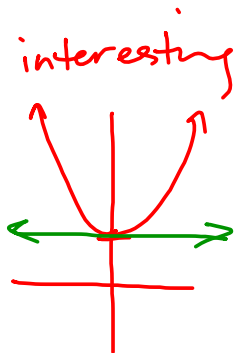
Use the Alternate Form of the Derivative:

Find the slope of the tangent line to the graph of $f(x) = x^2 + 1$ at the point $(0, 1)$

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

Find the slope of the tangent line to the graph of $f(x) = x^2 + 1$ at the point $(0, 1)$

$$\begin{aligned} @ (0, 1) \rightarrow & \lim_{x \rightarrow 0} \frac{x^2 + 1 - 1}{x - 0} \\ & \lim_{x \rightarrow 0} x = 0 \end{aligned}$$



Where the derivative is zero, you have a horizontal tangent.

$$\text{If } f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c} = \pm \infty ,$$

then it is a vertical tangent line!

The limit DNE, so f is not differentiable there.

If f is differentiable at $x = c$, then f is continuous at $x = c$

Use the Limit Definition:

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Find $f'(x)$ for $f(x) = \sqrt{x}$, and use the result to find the slope of f at $(1, 1)$, $(4, 2)$ and $(0, 0)$

Find $f'(x)$ for $f(x) = \sqrt{x}$, and use the result to find the slope of f at $(1, 1)$, $(4, 2)$ and $(0, 0)$

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x - x}{\Delta x (\sqrt{x+\Delta x} + \sqrt{x})} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} \\
 &= \frac{1}{\sqrt{x+0} + \sqrt{x}} \\
 &= \frac{1}{2\sqrt{x}} \leftarrow \text{slope}
 \end{aligned}$$

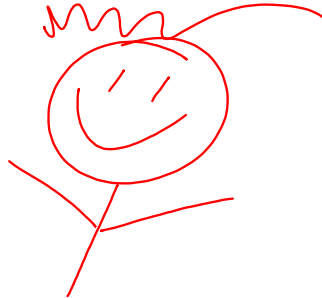
@ $(1, 1)$ $m = \frac{1}{2(1)} = \frac{1}{2}$
 @ $(4, 2)$ $m = \frac{1}{2\sqrt{4}} = \frac{1}{4}$
 @ $(0, 0)$ $m = \frac{1}{2\sqrt{0}} = \frac{1}{0} = \infty$
 So vertical tangent @ $(0, 0)$ slope is undefined.

Review for Ch. 2 Test:

- Strategies for finding limits
- One sided limits
- Equations for lines tangent to a curve
- Continuity
- The Intermediate Value Theorem
- Infinite Limits
- Vertical Asymptotes

(There will be a calculator part and a non-calculator part.)

Review Activity: Match 2 cards together based on the blue graph.



HW:

Review for Ch. 2

Test tomorrow!

Friday's
assignment:
p. 103 # 7-25 odd
and # 43